Test of preparedness to take Math 110A, Math 109, Math 107, Math 120, or Math 125

If you wish to take Math 110A, 109, 107, 120, or 125, you should work through the following problems (without a calculator; use pencil and paper only), and then check your answers against the ones provided. Students who are well-prepared to take one of these courses should get at least 10 of the 15 problems below (counting lettered sub-problems as separate problems), completely correct.

1. Simplify the following:
   
   \( (a) \frac{(2a^2b^{-3})^2}{(2a^{-1}b^0)^{-3}}, a \neq 0, b \neq 0 \) \n   \( (b) \frac{1}{x^2} - \frac{1}{y^2} \frac{2x^2 + 3xy + y^2}{xy} \)

2. Solve for \( x \):
   
   \( (a) \sqrt{x + 16} + 4 = x \) \n   \( (b) \frac{5}{2x + 1} + \frac{7}{x - 1} = \frac{21}{2x^2 - x - 1} \) \n   \( (c) 3x^3 - x^2 = 4x \)

3. Find the line perpendicular to \( 3x - 2y = 7 \) that goes through \((2, -1)\).

4. Given \( f(x) = \frac{1}{x - 1} \), find and simplify \( \frac{f(a + h) - f(a)}{h} \). (You may assume \( h \neq 0 \)).

5. Find \((f \circ g)(x)\) for \( f(x) = \frac{11x}{6 - x} \) and \( g(x) = 3x + 2 \). State the domain.

6. Sketch the graphs of the following:
   
   \( (a) f(x) = -2x^2 + 8x - 9 \)
   \( (b) g(x) = 3^{-x^2+1} \)
   \( (c) h(x) = \log_2(x + 2) \)
   \( (d) y = 3 \sin x - 2 \) from \(-2\pi\) to \(2\pi\)

7. Solve for \( x \):
       
   \( (a) \frac{1}{2x - 3x} = 32^x \)
   \( (b) \log_2(x + 2) + \log_2(x - 5) = 3 \)
   \( (c) \log_3 \frac{1}{27} = x \)
Answers

1. 
   (a) \( \frac{32a}{by} \)
   (b) \( \frac{y - x}{xy^2 + 2x^2y} \)

2. 
   (a) \( x = 9 \)
   (b) no solution
   (c) \( x = -1, 0, \) or \( 4/3 \)

3. \( 2x + 3y = 1 \)

4. \( \frac{-1}{(a - 1)(a + h - 1)} \)

5. \( (f \circ g)(x) = \frac{33x + 22}{4 - 3x} \), Domain = \( \{x \in \mathbb{R} | x \neq 4/3\} \)

6. 
   (a) 
   (b)
   (c) 
   (d)